

# MeshFree Benchmark Series

## 免網格分析軟體-基準測試

# Verification of Linear Static

# 線性靜態驗證

# 01 Straight Cantilever Beam

## 問題定義

A straight cantilever beam (Fig. VLS01, a) is subjected to four different loads at the free end (Fig. VLS01, b):

1. extension
2. in-plane shear
3. out-of-plane shear
4. twisting

單位: IPS

## 材料屬性

楊氏係數  $E = 1e+7$  psi 泊松比  $\nu=0.3$

## 條件和結果

Loads are uniformly distributed along face of the beam (Fig. 1-1-2a, b, c) or along edges (Fig. 1-1-2d).

Find tip displacements in direction of loads.

## 參考

"A proposed standard set of problems to Test Finite Element Accuracy", by R.H. MacNeal and R.L. Harder, Finite Elements in Analysis and Design I, 1985, pp. 3-20.

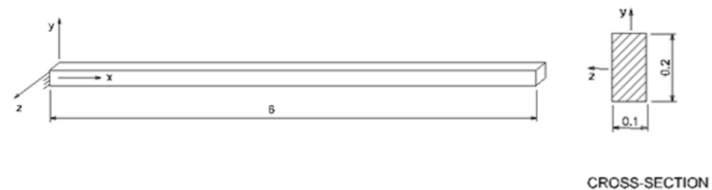


Fig. VLS01, a

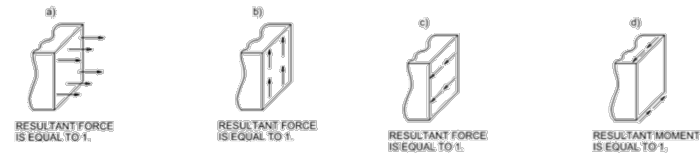
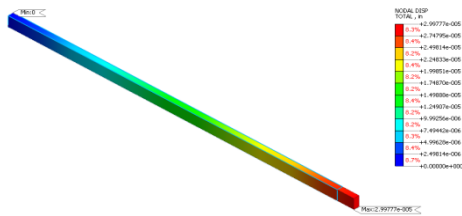
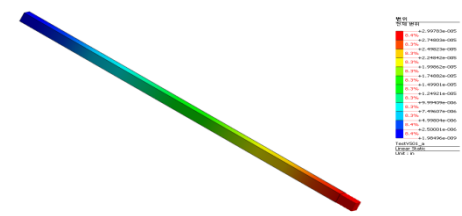
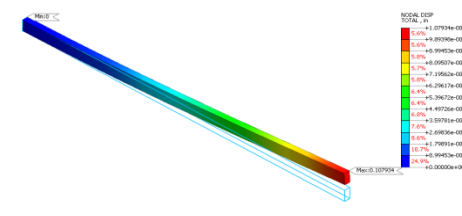
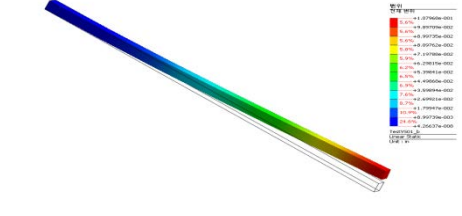


Fig. VLS01, b

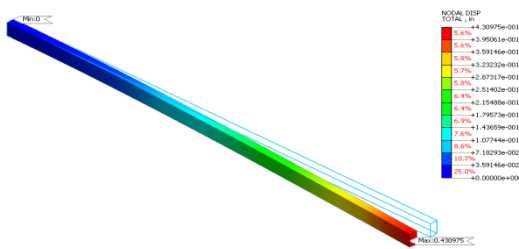
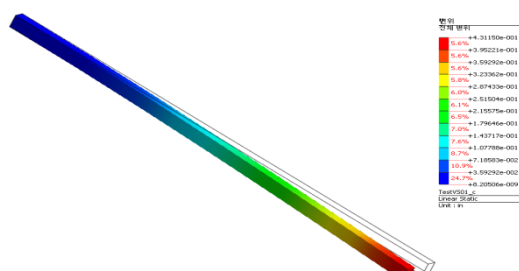
### Axial Loading (+x direction)

	端點-變形(in)	誤差 (%)
理論	3.0000E-05	-
FEM 有限元素 分析軟體	2.9978E-05	-0.07%
		
MeshFree 免網格式 分析軟體	2.9978E-05	-0.07%
		

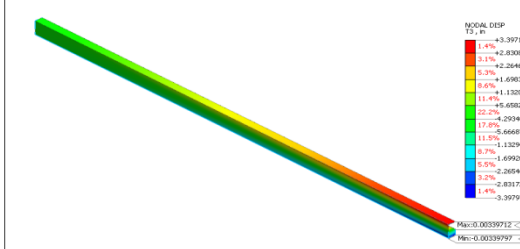
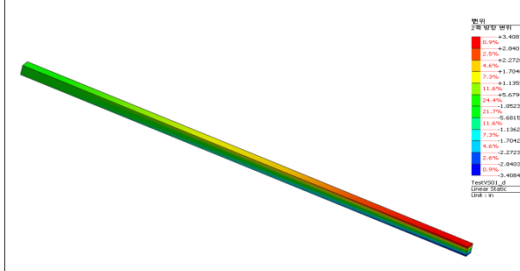
### In-Plane Shear (+y direction)

	端點-變形(in)	誤差 (%)
理論	1.0810E-01	-
FEM 有限元素 分析軟體	1.0793E-01	-0.02%
		
MeshFree 免網格式 分析軟體	1.0797E-01	-0.01%
		

## Out of Plane Shear

	端點-變形(in)	誤差(%)
理論	4.3210E-01	-
FEM 有限元素 分析軟體	4.3098E-01	-0.26%
		
MeshFree 免網格式 分析軟體	4.3115E-01	-0.02%
		

## Twisting

	端點-變形(in)	誤差(%)
理論	3.4080E-03	-
FEM 有限元素 分析軟體	3.3971E-03	-0.32%
		
MeshFree 免網格式 分析軟體	3.4082E-03	0.01%
		

# 02 Curved Veam

## 問題定義

A curved beam, spanning a 90 degree arc is fixed at one end and free at the other (Fig. vs02, a). The beam is subjected to two different loads at the free end (Fig. vs02, b):

1. in-plane load
2. out-of-plane load

單位: IPS.

## 材料屬性

楊氏係數  $E = 1e+7$  psi, 泊松比  $\nu=0.3$

## 條件和結果

Find tip displacements in direction of loads.

## 參考

"A proposed standard set of problems to Test Finite Element Accuracy", by R.H. MacNeal and R.L. Harder, Finite Elements in Analysis and Design I, 1985, pp. 3-20.

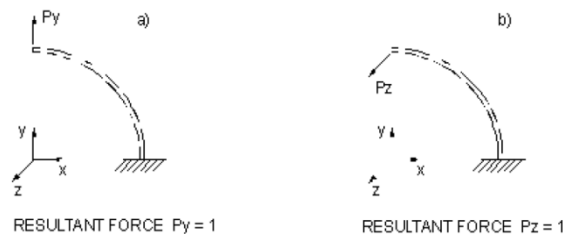
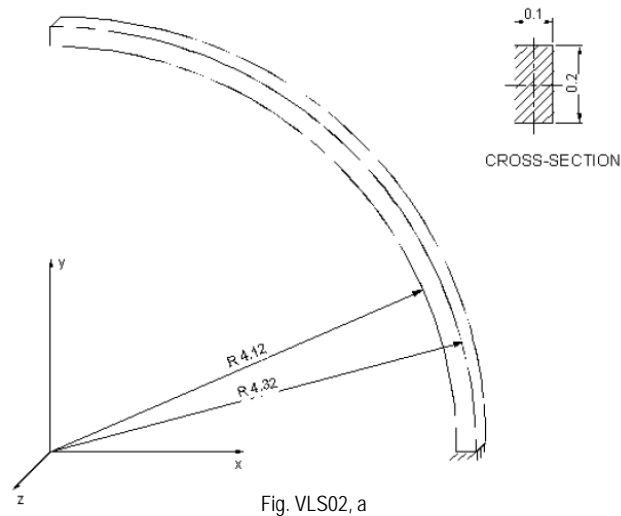
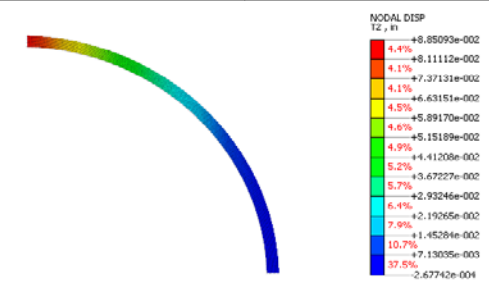
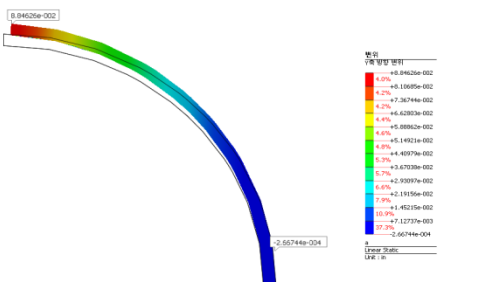
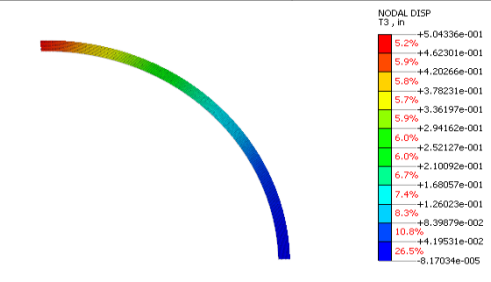



Fig. VLS02, b

## In-Plane Shear

	端點-變形(in)	誤差(%)
理論	8.7340E-02	-
FEM 有限元素 分析軟體	8.8509E-02	1.34%
		
MeshFree 免網格式 分析軟體	8.8463E-02	1.29%
		

## Out of plane Shear

	端點-變形(in)	誤差(%)
理論	5.0220E-01	-
FEM 有限元素 分析軟體	5.0434E-01	0.43%
		
MeshFree 免網格式 分析軟體	5.0423E-01	0.40%
		

# 03 Simply Support Thin Plate

## 問題定義

A square plate simply supported on four edges (Fig. VLS03, a) is subjected to the following loads (Fig. VLS03, b):

1. uniform pressure
2. concentrated load

單位: IPS

## 材料屬性

楊氏係數  $E=1.7472e7$  psi, 泊松比  $\nu=0.3$

## 條件和結果

solid model is used, boundary conditions on supported edges are applied along middle lines of these edges (Fig. VLS03, c).

Find tip displacements in direction of loads.

## 參考

"A proposed standard set of problems to Test Finite Element Accuracy", by R.H. MacNeal and R.L. Harder, Finite Elements in Analysis and Design I, 1985, pp. 3-20.

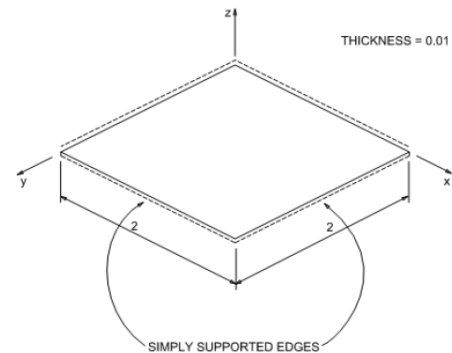


Fig. VLS03, a

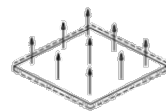


Fig. VLS03, b

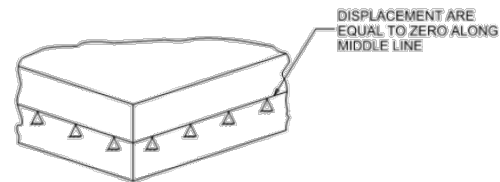
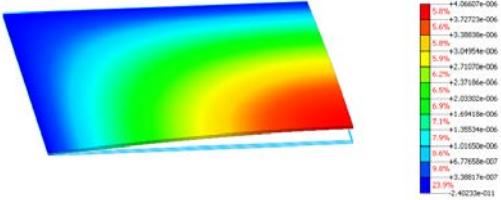
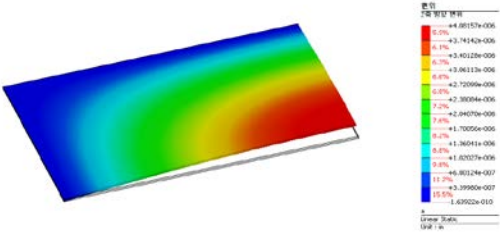


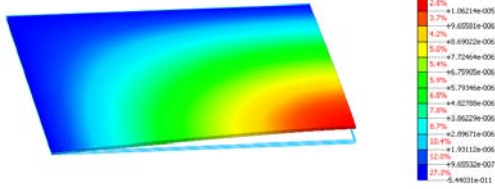
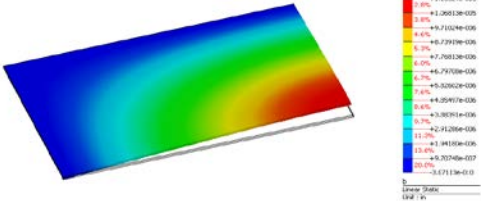
Fig. VLS03, c



## Uniform Load

	端點-變形(in)	誤差(%)
理論	4.0620E-06	-
FEM 有限元素 分析軟體	4.0661E-06	0.10%
		
MeshFree 免網格式 分析軟體	4.0816E-06	0.48%
		

## Point Load

	端點-變形 (in)	誤差(%)
理論	1.1600e-05	-
FEM 有限元素 分析軟體	1.1587e-05	-0.11%
		
MeshFree 免網格式 分析軟體	1.1652e-05	0.45%
		

# 04 Clamped Thin Plate

## 問題定義

A square plate clamped on four edges (Fig. VLS04, a) is subjected to the following loads (Fig. VLS04, b):

1. uniform pressure
2. concentrated load

單位: IPS.

## 材料屬性

楊氏係數  $E=1.7472e7$  psi 泊松比  $\nu=0.3$

## 條件和結果

Find tip displacements in direction of loads.

## 參考

"A proposed standard set of problems to Test Finite Element Accuracy", by R.H. MacNeal and R.L. Harder, Finite Elements in Analysis and Design I, 1985, pp. 3-20.

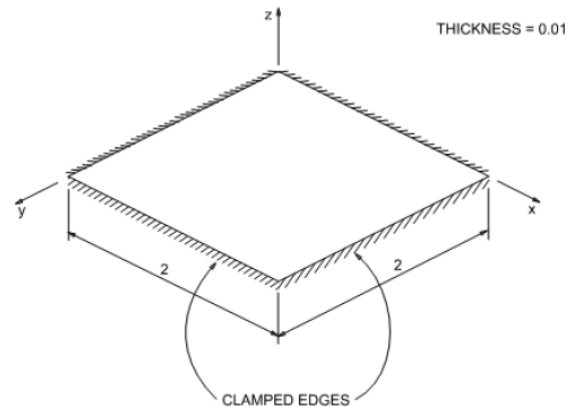


Fig. VLS04, a

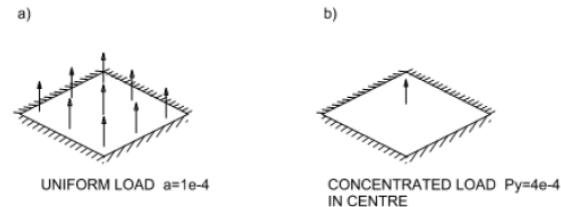


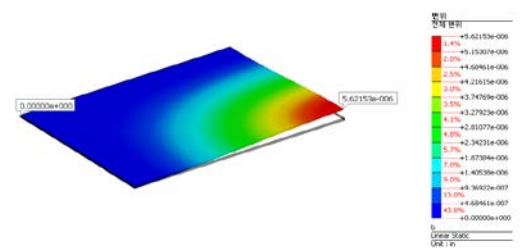
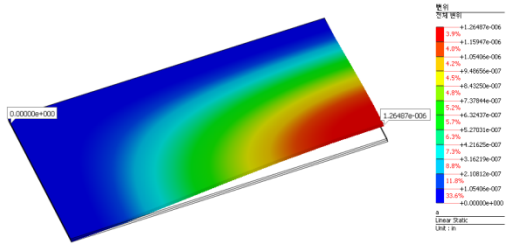
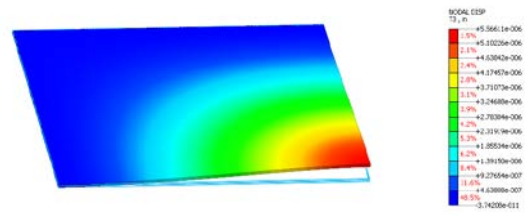
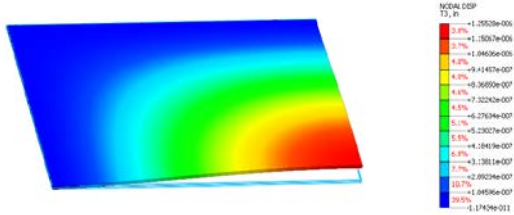
Fig. VLS04, b

## Uniform Load

	端點-變形(in)	誤差(%)
理論	1.2600e-06	-
FEM 有限元素 分析軟體	1.2553e-06	-0.37%
MeshFree 免網格式 分析軟體	1.2649e-06	0.39%

## Point Load

	端點-變形(in)	誤差(%)
理論	5.6000e-06	-
FEM 有限元素 分析軟體	5.5661e-06	0.61%
MeshFree 免網格式 分析軟體	5.6215e-06	0.38%



# 05 Cantilever Plate

## 問題定義

Find bending stress at fixed end for a cantilevered plate subjected to a vertical shear load of 120 lbs.  
(Fig. VLS05).

單位: IPS

## 材料屬性

楊氏係數  $E=10.7 \text{ e6 psi}$  泊松比  $\nu=0.3$

## 條件和結果

Load is expected to be uniformly distributed along free end of the plate.

## 參考

Ferdinand L., Singer, STRENGTH OF MATERIALS, (Harper & Row, 1962), Art. 52, p. 133.

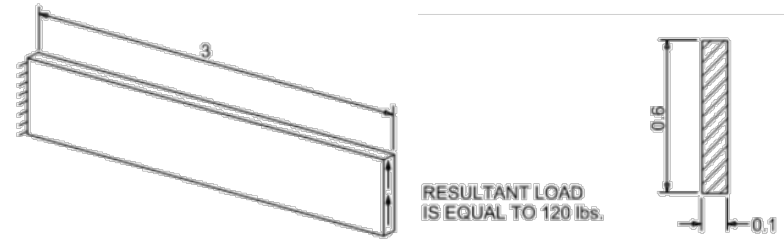
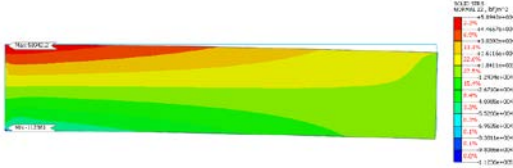
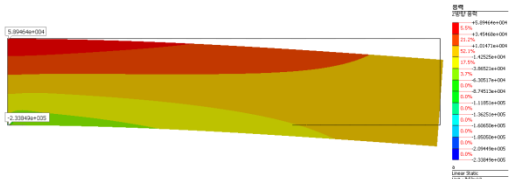
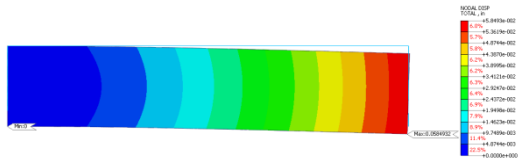
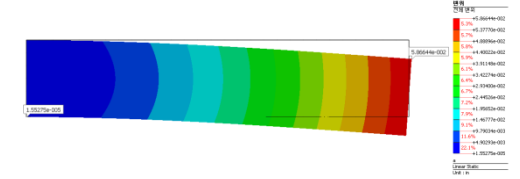


Fig. VLS05

## Maximum Bending Stress

	彎曲應力(psi)	誤差(%)
理論	6.0000e+04	-
FEM 有限元素 分析軟體	5.8942e+04	-1.76%
		
MeshFree 免網格 分析軟體	5.8946e+04	-1.76%
		

## Maximum Displacement of Tip

	端點-變形 (in)	誤差(%)
理論		
FEM 有限元素 分析軟體	5.8493e-02	-
		
MeshFree 免網格 分析軟體	5.8664e-02	0.29%
		

# 06 Plate with a Hole

## 問題定義

Plate with a hole. Due to symmetry, only 1/4 section was analyzed as shown.

單位: IPS

## 材料屬性

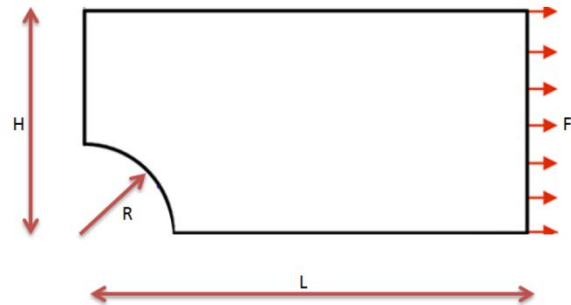
楊氏係數  $E=2.1 \times 10^{11}$  Pa, 泊松比  $\nu=0.3$

## 邊界和負載

Load  $F=100$  MPa, +x direction

## 參考

"A proposed standard set of problems to Test Finite Element Accuracy", by R.H. MacNeal and R.L. Harder, Finite Elements in Analysis and Design I, 1985, pp. 3-20.



$L=100$  mm,  $H=50$  mm,  $R=20$  mm Thickness=1 mm,

Fig. VLS06

## Reference Solution



$$\sigma_{max} = k\sigma_{nom} \quad \text{with} \quad k = 3$$

$$\sigma_{nom} = F \frac{h}{h-r} = 166.7 \text{ MPa}$$


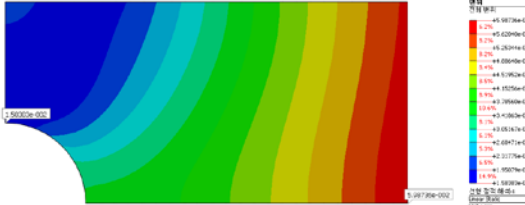
$$k = 3 - 3.13\left(\frac{d}{h}\right) + 3.66\left(\frac{d}{h}\right)^2 - 1.53\left(\frac{d}{h}\right)^3 = 2.2357$$

$$\sigma_{max} = 372.7 \text{ MPa}$$

## Maximum Stress

	X Axial Stress(MPa)	誤差(%)
理論	372.70	-
FEM 有限元素 分析軟體	377.35	1.25%
		
MeshFree 免網格 分析軟體	373.72	0.27%
		

## Maximum Displacement

	端點-變形 (in)	誤差(%)
理論		
FEM 有限元素 分析軟體	5.9899e-02	-
		
MeshFree 免網格 分析軟體	5.9874e-02	-0.04%
		

# 07 Scordelis-Lo Roof

## 問題定義

Curved cylindrical dome. Supported on curved edges ( $U_x, U_z = 0$ ), unconstrained on straight edges. Due to symmetry, only 1/4 section was analyzed as shown.

單位: SI

## 材料屬性

楊氏係數  $E=4.32e+8$  Pa, 泊松比  $\nu=0.0$

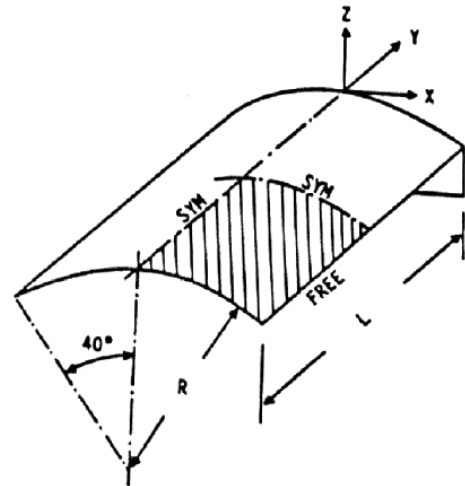
## 條件和結果

Load is -90 Pa at -Z Direction

Area of top face is  $2 * \pi * (R + .125) * (40/360) * 25 = 438.514 \text{ m}^2$ . To get -90 N/m<sup>2</sup>, need a total load of  $438.514 * 90 = 39466.25704 \text{ N}$

## 參考

The theoretical solution value of the Z displacement at the middle of free edge is given as 0.3086 though many finite element codes converge to the smaller value of 0.3024.

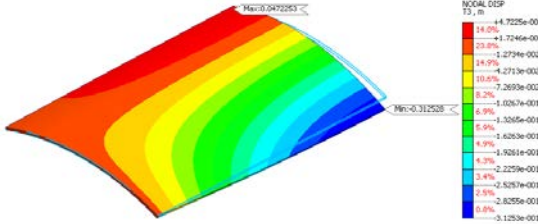
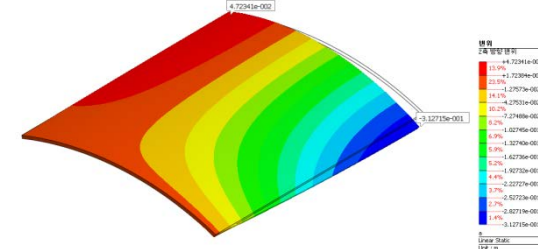


$$L = 50 \text{ m } R = 25 \text{ m } t = 0.25 \text{ m}$$

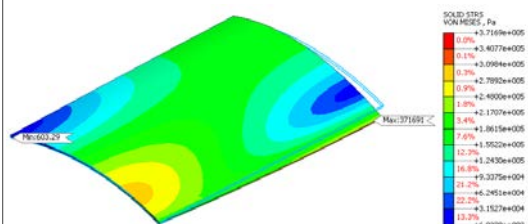
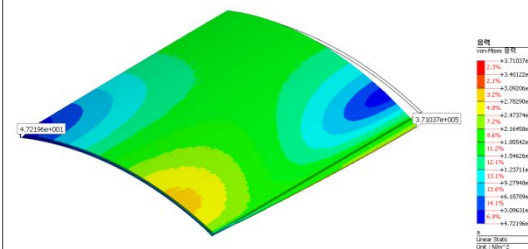
Fig. VLS07



## Maximum Z Displacement

	Z方向- 變形(m)	誤差(%)
理論	0.3086	-
FEM 有限元素 分析軟體	0.3125	1.26%
		
MeshFree 免網格式 分析軟體	0.3127	1.33%
		

## Maximum Von-mises Stress

	Von-mises stress(Pa)	誤差(%)
理論		
FEM 有限元素 分析軟體	3.7169e+5	-
		
MeshFree 免網格式 分析軟體	3.7104e+5	-0.17%
		

# 08

# Curved Thick Elliptical Plate

## 問題定義

For thick doubly curved beam, find tip displacement at free end. Reference Solution is 4.9352 in

單位: IPS

## 材料屬性

楊氏係數  $E=210e+09$  Pa, 泊松比  $\nu=0.3$

## 條件和結果

Load is uniform  $1.0e+06$  Pa, Pressure on face CDAB

Constraints are below

- slider constraint applied to faces  $ABB'A'$  and  $CDD'C'$
- $U_x$  and  $U_y = 0$  on face  $CBB'C'$
- $U_z = 0$  on line spot  $EE'$

Find  $S_{yy}$  Stress at vertex D. Reference Solution is 5.38 MPa

## 參考

Test LE10 from NAFEMS Publication TNSB, Rev 3, "The Standard NAFEMS Benchmarks", October 1990.

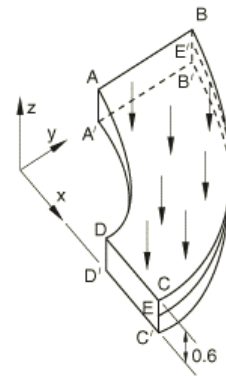
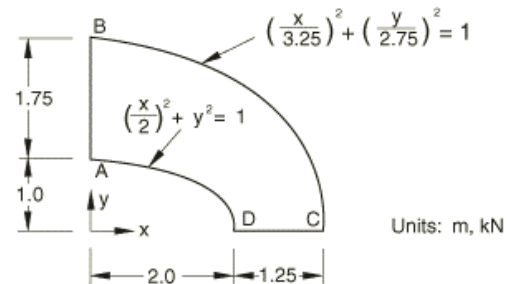
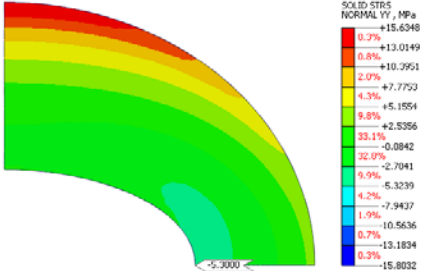
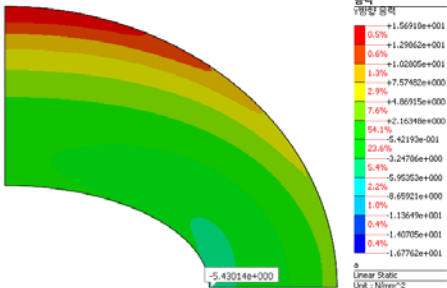

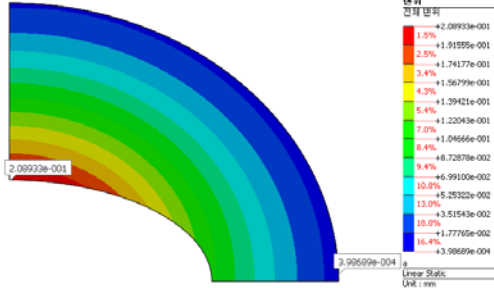


Fig. VLS08

## Syy Stress at Point D

	應力(MPa)	誤差(%)
理論	-5.380	-
FEM 有限元素 分析軟體	-5.300	-1.49%
		
MeshFree 免網格 分析軟體	-5.430	0.93%
		

## Maximum Displacement

	變形(mm)	誤差(%)
理論		
FEM 有限元素 分析軟體	0.207	-
		
MeshFree 免網格 分析軟體	0.209	0.97%
		

# 09 Raasch Challenge

## 問題定義

For thick doubly curved beam, find tip displacement at free end. Reference Solution is 4.9352 in

單位: IPS

## 材料屬性

楊氏係數  $E=3300\text{psi}$ , 泊松比  $\nu=0.3$

## 條件和結果

Load is uniform 1.0 lbf out of plane.

## 參考

Testing of Shell Elements using Challenging Benchmark Problems, F.T. Wong

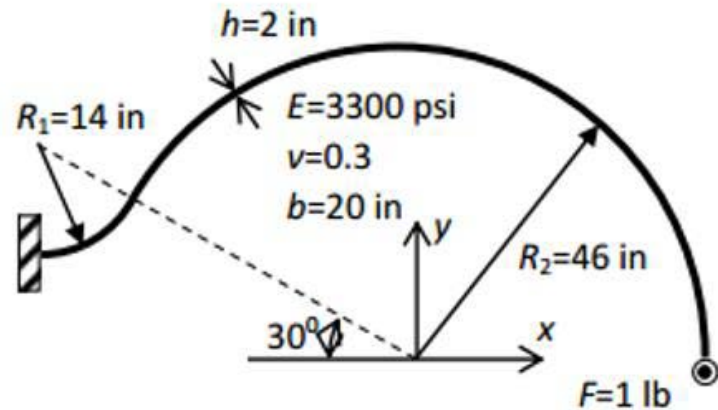
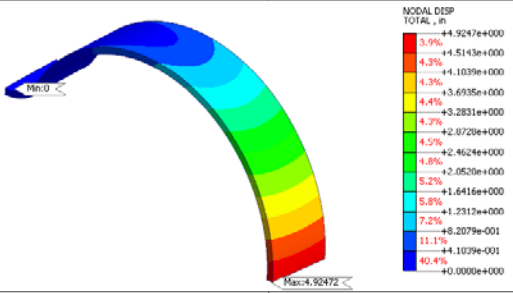
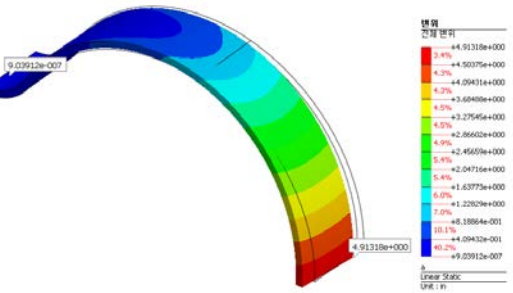
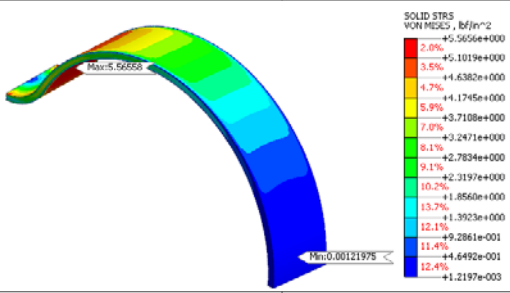
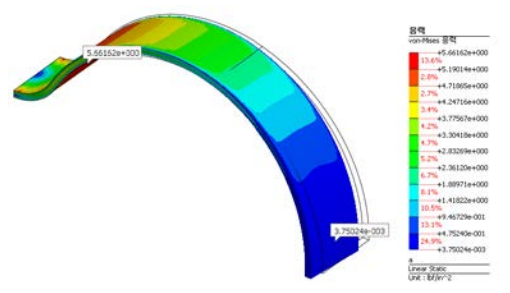


Fig. VLS09

## Displacement Magnitude

	端點-變形t(in)	誤差(%)
理論	4.9352	-
FEM 有限元素 分析軟體	4.9247	-0.21%
		
MeshFree 免網格式 分析軟體	4.9132	-0.45%
		

## Maximum Von-mises Stress

	Von-mises Stress(psi)	誤差(%)
理論		
FEM 有限元素 分析軟體	5.5656e+0	-
		
MeshFree 免網格式 分析軟體	5.6616e+0	1.72%
		

# 10 Skew Plate Under Pressure

## 問題定義

A skew plate (Fig. VLS10) is subjected to uniform pressure 0.7 KPa. Plate is simply supported at its perimeter:  $U_z = 0$  along edges AB, BC, CD, and AD.  $U_x = U_y = 0$  at point A and  $U_y = 0$  at point B to prevent rigid body motion.

單位: SI

## 材料屬性

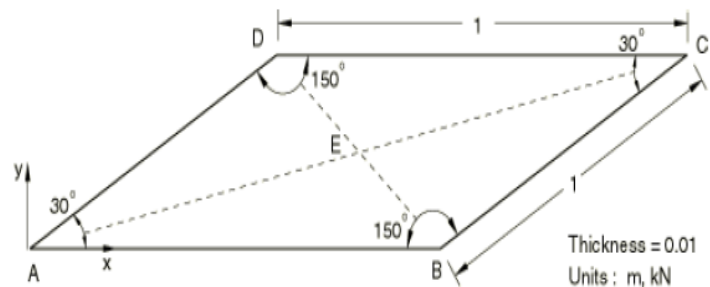
楊氏係數 =  $2.1 \times 10^{11}$  Pa, 泊松比 = 0.3

## 條件和結果

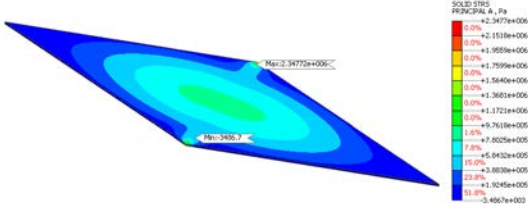
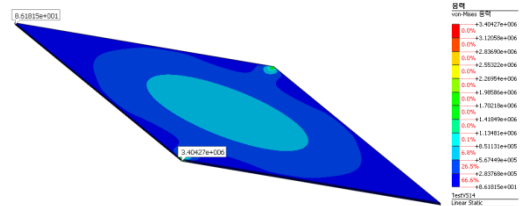
Find maximum principle stress at the plate center at point E.

## 參考

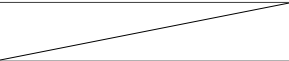
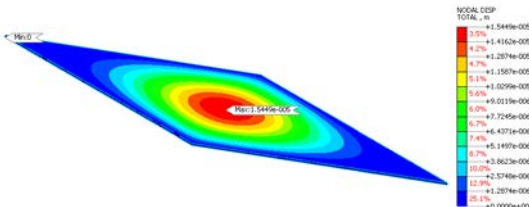
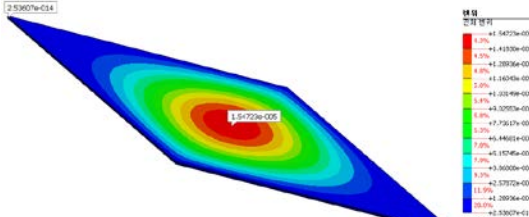
Test LE6 from NAFEMS Publication TNSB, Rev. 3, "The Standard NAFEMS Benchmarks," October 1990.



## Maximum Principle Stress

	主應力(MPa)	誤差(%)
理論	0.802	-
FEM 有限元素 分析軟體	0.821	2.37%
		
MeshFree 免網格式 分析軟體	0.822	2.49%
		

## Maximum Displacement

	變形(m)	誤差(%)
理論		-
FEM 有限元素 分析軟體	1.5449e-05	-
		
MeshFree 免網格式 分析軟體	1.5472e-05	0.15%
		

# 11

# Stress Concentration of Filleted Bar

## 問題定義

Two configurations of a filleted bar are loaded as shown.

單位: SI

## 材料屬性

楊氏係數 = 2.1e+11 Pa 泊松比= 0.3

## 條件和結果

Axial load : -1 MPa Pressure

Symmetry (sliding) boundary condition at bar center (left face opposite fillet in Fig. VLS 11)

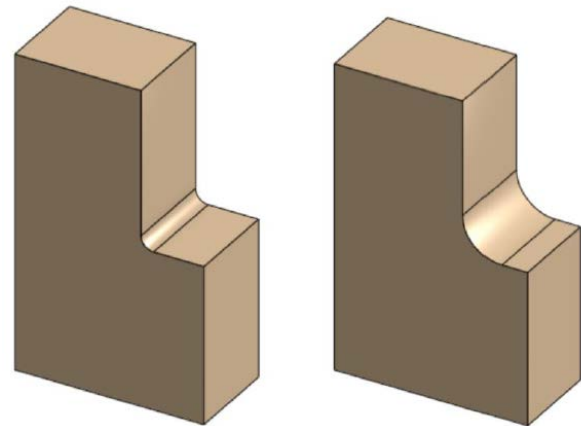
Find maximum priciple stress

## 參考

Shigley's Mechanical Engineering Design, Appendix A, Figure A-15-5, McGraw Hill, 2016

Reference Kt values are approximately 2.6 for configuration 1 and 1.9 for configuration 2. Stress concentration given by:

$$\sigma = Kt * \sigma_o$$

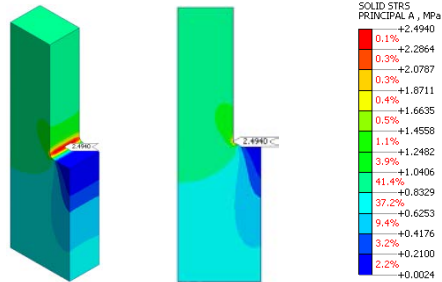
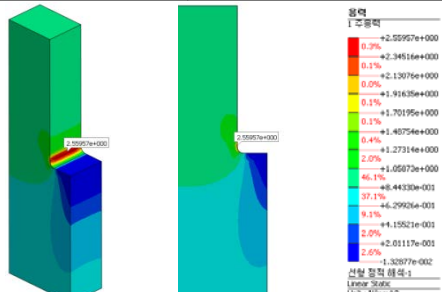


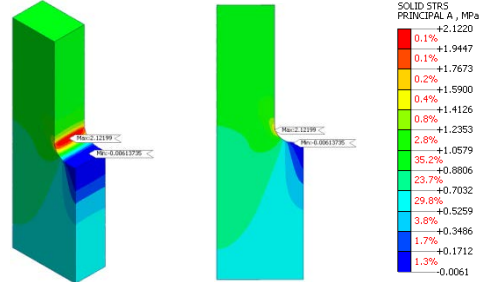
Dimension	Configuration 1	Configuration 2
D	60 mm	63 mm
d	40 mm	42 mm
r	4 mm	12.6 mm
T	30 mm	30 mm
D/d	1.50	1.50
r/d	0.05	0.15

Fig. VLS11



## Maximum Principle Stress

	主應力(MPa)	誤差(%)
理論	2.58	-
FEM 有限元素 分析軟體	2.49	-3.49%
		
MeshFree 免網格 分析軟體	2.56	2.49%
		

	應力(MPa)	誤差(%)
理論	2.07	-
FEM 有限元素 分析軟體	2.12	2.42%
		
MeshFree 免網格 分析軟體	2.02	2.42%
	